# Worksheet for November 22

Problems marked with an asterisk are to be placed in your math diary.

(1.*) The Divergence Theorem of Gauss, in rectangular coordinates, states the following: Suppose $S$ is a closed surface bounding the solid $B$. Let $\mathbf{F} = F_1\vec{i} + F_2\vec{j} + F_3\vec{k}$ be a vector field defined on $S$ whose component functions are continuously differentiable. Then

$$\int\int_S \mathbf{F} \cdot d\mathbf{S} = \int\int\int_B \left(\frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}\right) dV.$$

Verify Gauss's theorem for $\mathbf{F} = x\vec{i} + y\vec{j} + z\vec{k}$ and $S$ the sphere of radius $R$ centered at the origin.

(2.*) Stoke's Theorem, in rectangular coordinates states, the following: Suppose the closed curve $C$ forms the boundary of a surface $S$. Let $\mathbf{F} = F_1\vec{i} + F_2\vec{j} + F_3\vec{k}$ be a vector field defined on $S$ whose component functions are continuously differentiable. Suppose further that as one traverses $C$, the surface is to the left. Then

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int\int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S},$$

where $\nabla \times \mathbf{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$. Verify Stoke's theorem for $\mathbf{F} = (2z - y)\vec{i} + (x + z)\vec{j} + (3x - 2y)\vec{k}$ and $S$ the paraboloid $z = 9 - x^2 - y^2$ defined over the disk in the $xy$-plane of radius 3. Note that the boundary of $S$ is the circle of radius 3 in the $xy$-plane centered at the origin. Be sure to paramterize $S$ to get an upward pointing normal vector.

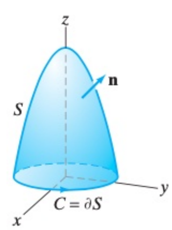


**Figure 7.31** The paraboloid $z = 9 - x^2 - y^2$ oriented with upward normal **n**. Note that the boundary circle $C$ is oriented consistently with $S$.

**Bonus Problem 8.** Let $S_\epsilon$ denote the sphere of radius $\epsilon$ centered at the point $(x_0, y_0, z_0) \in \mathbb{R}^3$ and $\mathbf{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$. Show that

$$\lim_{\epsilon \to 0} \frac{1}{\text{vol}(S_\epsilon)} \int\int_{S_\epsilon} \mathbf{F} \cdot d\mathbf{S} = 2x_0 + 2y_0 + 2z_0.$$

This problem is worth 5 points and is due at the start of class on Monday, November 25. You may use tables of integrals to solve this problem.